Problem 6.4-6

a. We find \( \mu \) and \( \sigma \) as follows:

\[
\mu = \int_0^2 x \left( 1 - \frac{x}{2} \right) \, dx
\]

\[
= \frac{2}{3}
\]

\[
\sigma^2 = \int_0^2 x^2 \left( 1 - \frac{x}{2} \right) \, dx - \mu^2
\]

\[
= \int_0^2 x^2 \left( 1 - \frac{x}{2} \right) \, dx - \left( \frac{2}{3} \right)^2
\]

\[
= \frac{2}{3} - \left( \frac{2}{3} \right)^2
\]

\[
= \frac{2}{9}
\]

b. To find the given probability we proceed as follows:

\[
P \left( \frac{2}{3} \leq \bar{X} \leq \frac{5}{6} \right) = P \left( \frac{\frac{2}{3} - \mu}{\sigma/\sqrt{n}} \leq Z \leq \frac{\frac{5}{6} - \mu}{\sigma/\sqrt{n}} \right)
\]

substituting for \( \mu \) and \( \sigma \) from part a), with \( n = 18 \) we get

\[
= P(0 \leq Z \leq 1.5)
\]

\[
= 0.4332
\]

Problem 6.4-8

a. \( E(X) = \mu = 24.43 \)

b. \( \text{Var}(X) = \sigma^2/n = 2.20/30 = 0.0733 \)

c. To find the given probability we proceed as follows:

\[
P(24.17 \leq \bar{X} \leq 24.82) \approx P \left( \frac{24.17 - \mu}{\sigma/\sqrt{n}} \leq X \leq \frac{24.82 - \mu}{\sigma/\sqrt{n}} \right)
\]

\[
= P \left( \frac{24.17 - 24.43}{\sqrt{0.0733}} \leq X \leq \frac{24.82 - 24.43}{\sqrt{0.0733}} \right)
\]

\[
= P(-0.96 \leq Z \leq 1.44)
\]

\[
= 0.7566
\]