Problem 3.2-5

Let $X$ equal the value of the prize. Then the expected value of this game to me is

\[
E(X - 0.50) = E(X) - 0.50 = \frac{12000}{3000000}(25) + \frac{4}{300000}(10000) + \frac{1}{3000000}(50000) + \frac{1}{3000000}(200000) + \frac{300000 - 12006}{3000000}(0) - 0.50
\]

\[
= \frac{59}{300} - 0.50
\]

\[
= -\frac{91}{300}
\]

\[
= -30.33
\]

This lottery game would be worth -33.33 cents a week to me.

Problem 3.2-7

Let $X$ be the expected payoff. Suppose you win $1, then $X(1) = 1 + 1 - 1 = 1$. You win a dollar, you get your dollar back, and you are out the dollar you bet. Therefore, the payoff to you is $1. On the other hand, $X(0) = -1$ since you have lost a dollar in betting, and since you didn’t win you don’t get your $1 back.

Thus,

\[
X(0) = -1, \quad f(-1) = \frac{125}{216}
\]

\[
X(1) = 1, \quad f(1) = \frac{75}{216}
\]

\[
X(2) = 2, \quad f(2) = \frac{15}{216}
\]

\[
X(3) = 3, \quad f(3) = \frac{1}{216}
\]

So,

\[
E(X) = \frac{125}{216}(-1) + \frac{75}{216}(1) + \frac{15}{216}(2) + \frac{1}{216}(3)
\]

\[
= -\frac{17}{216}
\]

\[
= -0.0787
\]

The expected payoff for this game is a loss of $0.0787 per dollar bet.

Problem 3.2-13

Let $X$ be the value of the winnings. Then

\[
X(-1) = -1, \quad f(-1) = 0.50707
\]

\[
X(1) = 1, \quad f(1) = 0.49293
\]

Therefore,

\[
E(X) = 0.50707(-1) + 0.49293(1)
\]

\[
= -0.0141
\]
Problem 3.2-15

a. Solution:

\[ \mu = E(X) \]
\[ = \sum_{x \in S} xf(x) \]
\[ = 5 \left( \frac{1}{5} \right) + 10 \left( \frac{1}{5} \right) + 15 \left( \frac{1}{5} \right) + 20 \left( \frac{1}{5} \right) + 25 \left( \frac{1}{5} \right) \]
\[ = 15 \]

\[ \sigma^2 = Var(X) \]
\[ = E(X^2) - [E(X)]^2 \]
\[ = \sum_{x \in S} xf(x) - [E(X)]^2 \]
\[ = 5^2 \left( \frac{1}{5} \right) + 10^2 \left( \frac{1}{5} \right) + 15^2 \left( \frac{1}{5} \right) + 20^2 \left( \frac{1}{5} \right) + 25^2 \left( \frac{1}{5} \right) - 15^2 \]
\[ = 50 \]

b.

\[ \mu = E(X) \]
\[ = \sum_{x \in S} xf(x) \]
\[ = 5(1) \]
\[ = 5 \]

\[ \sigma^2 = Var(X) \]
\[ = E(X^2) - [E(X)]^2 \]
\[ = \sum_{x \in S} xf(x) - [E(X)]^2 \]
\[ = 5^2(1) - 5^2 \]
\[ = 0 \]

c.

\[ \mu = E(X) \]
\[ = \sum_{x \in S} xf(x) \]
\[ = 1 \left( \frac{3}{6} \right) + 2 \left( \frac{2}{6} \right) + 3 \left( \frac{1}{6} \right) \]
\[ = \frac{5}{3} \]

\[ \sigma^2 = Var(X) \]
\[ = E(X^2) - [E(X)]^2 \]
\[ = \sum_{x \in S} xf(x) - [E(X)]^2 \]
\[ = 1^2 \left( \frac{3}{6} \right) + 2^2 \left( \frac{2}{6} \right) + 3^2 \left( \frac{1}{6} \right) - \left( \frac{5}{3} \right)^2 \]
\[ = \frac{5}{9} \]