Problem 2.4-3

a. \( P(A \cap B) = P(A)P(B) = \frac{1}{4} \cdot \frac{2}{3} = \frac{1}{6} \)

b. \( P(A \cap B') = P(A)[1 - P(B)] = \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12} \)

c. \( P(A' \cap B') = [1 - P(A)][1 - P(B)] = \frac{2}{3} \cdot \frac{1}{3} = \frac{1}{3} \)

d. \( P[(A \cup B)'] = P(A' \cap B') = [1 - P(A)][1 - P(B)] = \frac{3}{4} \cdot \frac{1}{4} = \frac{3}{16} \)

e. \( P(A' \cap B) = [1 - P(A)]P(B) = \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{4} \)

Problem 2.4-7

a. Solution:

\[
P(\text{exactly one is successful}) = P(A_1 \cap A_2' \cap A_3') + P(A_1' \cap A_2 \cap A_3') + P(A_1 \cap A_2' \cap A_3)
\]
\[
= P(A_1)[1 - P(A_2)][1 - P(A_3)] + [1 - P(A)]P(A_2)[1 - P(A_3)] + [1 - P(A)][1 - P(A_2)]P(A_3)
\]
\[
= (.5)(.3)(.4) + (.5)(.7)(.4) + (.5)(.3)(.6)
\]
\[
= 0.29
\]

b. Solution:

\[
P(\text{exactly two are successful}) = P(A_1 \cap A_2 \cap A_3') + P(A_1' \cap A_2 \cap A_3) + P(A_1 \cap A_2' \cap A_3)
\]
\[
= P(A_1)P(A_2)[1 - P(A_3)] + [1 - P(A)]P(A_2)P(A_3) + P(A_1)[1 - P(A_2)]P(A_3)
\]
\[
= (.5)(.7)(.4) + (.5)(.7)(.6) + (.5)(.3)(.6)
\]
\[
= 0.44
\]

Problem 2.4-9

a. Solution:

\[
P(A \cap B \cap C) = P(A)P(B)P(C) = (.5)(.8)(.9) = 0.36
\]

b. Solution:

\[
P(\text{exactly two are successful}) = P(A \cap B \cap C') + P(A' \cap B \cap C) + P(A \cap B' \cap C)
\]
\[
= P(A)P(B)[1 - P(C)] + [1 - P(A)]P(B)P(C) + P(A)[1 - P(B)]P(C)
\]
\[
= (.5)(.8)(.1) + (.5)(.8)(.9) + (.5)(.2)(.9)
\]
\[
= 0.49
\]

c. Solution:

\[
P(\text{none}) = P(A')P(B')P(C')
\]
\[
= (.5)(.2)(.1)
\]
\[
= .01
\]